Written Exam at the Department of Economics winter 2017-18

Microeconomics III

Resit Exam

Date: February 9, 2018

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 3 pages in total

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

PLEASE ANSWER ALL QUESTIONS.
PLEASE EXPLAIN YOUR ANSWERS.

1. Find all the pure and mixed strategy Nash Equilibria (NE) of the following game.

		Player 2		
		L	C	R
Player 1	U	$0, 1$	$1, 10$	$1, 2$
	D	$1, 0$	$0, 0$	$1, 1$

Note: To fix notation, let p be the probability with which player 1 plays U, let r be the probability with which player 2 plays L, and let q be the probability with which player 2 plays C.

2. Two tech entrepreneurs have made 1 dollar through a new app and need to decide how to allocate the gains. If they can't agree, nobody gets anything. Let x_1 and x_2 be the amounts that entrepreneur 1 and 2 get. Their payoffs are:

$$\begin{aligned} u_1(x_1) &= x_1^2 \\ u_2(x_2) &= x_2. \end{aligned}$$

   (a) Calculate U, the set of possible payoff pairs. Can the symmetry axiom (SYM) be used to conclude that the Nash Bargaining Solution must satisfy $v_1^* = v_2^*$? Why/why not? (1 sentence).

   (b) Find the Nash Bargaining Solution. What are the allocations? That is, how much money does each entrepreneur get?

   (c) Suppose now that the entrepreneurs have signed a contract such that in case of disagreement, entrepreneur 2 gets to keep 0.5 dollar whereas entrepreneur 1 gets nothing. What is the new disagreement point? Find the Nash Bargaining Solution. What are the allocations?

   (d) Compare the allocations in (c) to those in (b), and comment on any difference you find.

3. Suppose we are in a **private value** auction setting. There are two bidders, $i = 1, 2$. They have valuation v_1 and v_2, respectively. These values are distributed independently uniformly with

$$v_i \sim U(1, 2).$$

The auction format is **sealed-bid first price**. In case of a tie, a fair coin is flipped to determine the winner.

(a) Suppose player j uses the strategy $b(v_j) = cv_j + d$, where c and d are constants. Show that if bidder $i \neq j$ bids b_i, his probability of winning is

$$\mathbb{P}(i \text{ wins}|b_i) = \frac{b_i - d - c}{c},$$

whenever $c + d \leq b_i \leq 2c + d$.

Hint: Recall that if $x \sim U(a, b)$ then $\mathbb{P}(x \leq y) = \frac{y-a}{b-a}$ for $y \in [a, b]$.

(b) Using the result in (a), show that there is a symmetric Bayesian Nash equilibrium (BNE) in linear strategies $b(v_i) = cv_i + d$, $i = 1, 2$. Find c and d.

(c) Now suppose instead that we are in a **common value** auction setting. The auction format is still **sealed-bid first price**. Thus, the object has common value $v_1 = v_2 = v$. We assume that

$$v = 1 + s_1 + s_2,$$

where s_1 and s_2 are independently distributed according to

$$s_i \sim U(0, 1/2).$$

Bidder i observes only s_i, but not s_j. Show that there is a symmetric BNE in which both bidders use the strategy $b(s_i) = cs_i + d$, $i = 1, 2$, and find c and d.